The article presents a macroeconomic equilibrium model in which aggregate demand and aggregate supply are considered not in relation to the price level, as is traditionally done, but in terms of functions dependent on the average tax rate. The concepts of optimal and equilibrium tax rates are introduced. In the former case, aggregate supply is maximum, while in the latter case aggregate demand and supply coincide. Based on an analysis of the model, it is shown that when the government tries to maintain the equilibrium average tax rate at a fixed level, the optimal tax rate becomes dependent on the price level, and an appropriate change in aggregate demand may lead to approximation of the optimal rate to the equilibrium rate. It is also demonstrated that each given value of the equilibrium tax rate can be matched with a set of functions and curves of aggregate supply and the national budget’s tax revenues.

In contemporary economic theories, the role of taxes, unfortunately, is studied one-sidedly. Keynesianism mostly emphasizes the mechanisms by which taxes affect the economy through aggregate demand and almost does not take into account the mechanism of their effect on the aggregate supply side. In other words, Keynesian models consider a situation in which aggregate supply does not depend on taxes, while aggregate demand, especially the part of it that involves consumption, is a
function of taxes. The problem of taxes is also seen one-sidedly by supply-side economics, in which the effect of the tax rate on aggregate supply is brought to the fore. The role of taxes can be fully explained, and the one-sided nature of these two theories can be overcome through a synthesis of supply-side economics and Keynesianism, which was first suggested by one of the present authors.¹

The model presented below is a further development of this idea of a Laffer-Keynesian synthesis. It is based on a macroeconomic equilibrium model that consists of aggregate demand and aggregate supply functions. However, in contrast to the standard models of aggregate demand and aggregate supply, these functions are analyzed not in the coordinates of a plane on the axes of which values of the price level \( P \) and output \( Y \) are shown, but on a plane in which the vertical axis corresponds to the average tax rate \( t \); and the horizontal axis, to output \( Y \). In this model, the price level, together with the rest of the factors that affect aggregate demand and aggregate supply, is exogenously given.

**Model of aggregate demand**

We begin our analysis with an examination of aggregate demand functions, which we designate as \( Y^D(t) \). This notation indicates that aggregate demand \( Y^D \) is considered in the form of functions of the average tax rate \( t \). Based on a modified version of a simplified Keynesian model, it has been shown² that, if analysis is limited just to the product market, then the aggregate demand function that depends on the average tax rate can be expressed in the following form:

\[
Y^D(t) = \frac{\tilde{A}}{1 - (1 - t)b - tg},
\]

(1)

where \( \tilde{A} \) is planned autonomous spending; this parameter is the sum of autonomous household consumption \( a \), autonomous government purchases \( G_o \), gross domestic private investments \( I_0 \), and net exports \( NX_o \):

\[
\tilde{A} = a + \tilde{G}_0 + I_0 + NX_0;
\]

\( b \) is the marginal propensity of households to consume, \( 0 < b < 1 \); and \( g \) is the marginal propensity for government purchases, \( 0 \leq g \leq 1 \).

To give the aggregate demand function its final form, it is advisable to expand our analysis and, along with the product market, also look at the money market. For this purpose, we represent gross domestic investments \( I_o \), which are a part of autonomous spending, not in the form of an exogenous quantity, but in the form of a function that depends on the interest rate:³

\[
I_0 = \tilde{I}_0 - \mu i,
\]

where \( \tilde{I}_0 \) is the part of investment spending that does not depend on the interest rate or on the amount of output (based on this content, \( \tilde{I}_0 \) refers to autonomous investments); \( \mu \) is a nonnegative parameter characterizing the sensitivity of investments
to the interest rate; \( i \) is the nominal interest rate. The value of \( i \) is established in the money market by the mechanism of balancing the money supply and demand. In the simplest case, the equation corresponding to equilibrium of the money market can be represented in the following form:\(^5\)

\[
\frac{M}{P} = kY^D(t) - hi,
\]

where \( M/P \) is the real cash balance of money in circulation (it is determined by the ratio of the nominal amount of money \( M \) and the price level \( P \)); and \( k, h \) are positive coefficients expressing the sensitivity of the demand for money to aggregate spending \( Y^D(t) \) and the interest rate \( i \).

Taking the equations given above into account in (1) and making the appropriate transformations, we get the following model of aggregate demand:\(^6\)

\[
Y^D(t) = \frac{A}{1 - (1-t)b - tg + \mu k / h}.
\] (2)

Here the quantity \( A \) consists of exogenously given elements. In particular,

\[
A = a + \tilde{G}_0 + \tilde{I}_0 + NX_0 + \mu \left( \frac{M}{P} \right) = \tilde{A}_0 + \frac{\mu}{h} \left( \frac{M}{P} \right).
\] (3)

As we see, in addition to the traditional autonomous spending \( A_0 = a + \tilde{G}_0 + \tilde{I}_0 + NX_0 \), \( A \) also contains an element that is determined by the real cash balance of money \( (M/P) \).

It can be shown\(^7\) that the properties of function (2) are determined by the interrelation of the marginal propensity to consume \( b \) and the marginal propensity for government purchases \( g \). All else being equal, \( Y^D(t) \), in the area of its determination,\(^8\) is increasing in relation to \( t \) when \( g > b \), decreasing when \( g < b \), and indifferent when \( g = b \). A graphic illustration of the function \( Y^D(t) \) with various possible combinations of values of \( b \) and \( g \) is given in Figure 1. It should be noted that, all else being equal, a change in \( A \), which is determined from (3), shifts the curves corresponding to \( Y^D(t) \). In particular, with an increase in \( A \), the curves \( Y^D(t) \) shift upward in parallel, which for a given average tax rate means an increase in aggregate demand. On the other hand, with a decrease in \( A \) the curves \( Y^D(t) \) shift downward in parallel, which for a given average tax rate corresponds to a reduction in aggregate demand.

**Model of aggregate supply**

As the aggregate supply that depends on the average tax rate we can consider a function of total output that fully satisfies the conditions of the Laffer theory and has the following form:\(^10\)

\[
Y^S(t) = Y_{pot} f(t) = Y_{pot} (er^t\ln r),
\] (4)
where $Y_{pot}$ expresses the volume of potential output with full use of economic resources in conditions of the existing technology; $f(t) = -e^r \ln \delta$ is a function that reflects the institutional aspect and determines the cumulative tax effect on supply. This is a behavioral function; it shows the degree of use of existing economic resources with a given value of the average tax rate (i.e., in conditions of a given institutional environment). Based on this content, $f(t)$ should satisfy the condition $0 \leq f(t) \leq 1$. When $f(t)$ takes a value close to 1, this indicates that economic activity is high for the given average tax rate and the level of use of resources is close to the maximum. The opposite situation takes place when $f(t)$ is close to zero. In this function, $e$ represents the base of the natural logarithm, and $\delta$ is a positive parameter that we will discuss later.

Figure 1. Versions of Aggregate Demand Curves

![Aggregate Demand Curves Diagram](image-url)
We have investigated the properties of the aggregate supply function (4)11 and have shown that $Y^S(t)$ is increasing in the interval $[0, t^*)$, decreasing in the interval $(t^*, 1]$, and

$$
\lim_{t \to 0} Y^S(t) = 0, \quad \max Y^S(t) = Y^S(t^*) = Y_{pot},
$$

$$
t^* = \exp\left(-\frac{1}{\delta}\right) = e^{-1/\delta}, \quad Y^S(1) = 0.
$$

With an average tax rate $t^* = e^{-1/\delta}$, the institutional environment fosters full use of resources, and output is maximum, that is, it is at the potential level $Y_{pot}$. Consequently, $t^*$ can be considered the optimal average tax rate. A graphic illustration of function (4) is given in Figure 2 (the curve $Y^S$). It shows that when all else is equal, movement on the ascending part of the curve corresponds to a change in the average tax rate from 0 to $t^*$; and movement on the descending part of the curve $Y^S$, to a change from $t^*$ to 1.12

What happens if “all else being equal” changes? First we should point out that in the conditions of model (4) for aggregate supply, a change in all else being equal involves a change either in the amount of potential output ($Y_{pot}$) or in the value of parameter $\delta$. As a rule, a change in $Y_{pot}$ occurs mostly in the long term, and the reason for such a change may be an increase or decrease in labor or the existing amount of capital, improvement or deterioration in the quality of technologies, a change in labor productivity, and so on. When these changes have a positive effect on $Y_{pot}$, then the curve $Y^S(t)$ shifts upward on the coordinate plane (aggregate supply increases, i.e., with the existing average tax rate more products and services will be produced in the economy than there were before the increase in $Y_{pot}$). Obviously, a decrease in the amount of potential output will cause the opposite result: the curve $Y^S(t)$ will shift downward and, in conditions of the existing tax rate, aggregate supply will decrease. Interestingly, when all else being equal is unchanged, neither an increase nor a decrease in potential output affects the optimal value of average tax rate $t^*$. In other words, with a change in $Y_{pot}$ the maximum of function (4) changes, but the value of the tax rate at which this maximum is attained does not change.

A completely different situation occurs when a disruption of all else being equal in the aggregate supply model (4) involves a change in the current market conditions, price level, production costs per unit of output, or producers’ expectations. These circumstances, especially a change in the price level, do not have affect the amount of potential output $Y_{pot}$ or, accordingly the maximum value of the function $Y^S(t)$; however, they do affect parameter $\delta$ and the optimal tax rate determined by this parameter.13 In other words, for example, with an increase in the price level the maximum value of the function $Y^S(t)$ will not change; however, the value of the average tax rate at which this maximum is attained will change.14 Consequently, in this case, a graph of the function $Y^S(t)$ will shift (deviate) from the position $Y^S$ to the right or left. In Figure 2, the dashed curves $Y^S_1$ and $Y^S_2$ correspond to forms
of the shift (deviation). The specific direction of the shift (deviation) depends on which of the intervals \([0, t^*)\) or \((t^*, 1]\) the existing tax rate is in. If we assume that the current rate is in the interval \([0, t^*)\) and, for example, is \(t_1\) \((t_1 \in [0, t^*)\), then with an increase in the price level, the graph of the function \(Y^S(t)\) will shift to the position \(Y^S_1\), while if we take \(t_2\) as the existing rate, which is in the interval \((t^*, 1]\), that is, \(t_2 \in (t^*, 1]\), then the shift (deviation) will be to the right, to the position \(Y^S_2\). The standard premise known from economics courses—that if there is not full employment, an increase in the price level (or a decrease in production costs per unit of production, or improvement of the current market conditions, etc.) has a positive effect on aggregate supply and causes it to increase—is fulfilled in the process of these changes.

**Macroeconomic equilibrium model**

Using the aggregate demand models (2)–(3) given above and the aggregate supply model (4), we will write the condition of macroeconomic equilibrium \(Y^S(t) = Y^D(t)\), determined in relation to the average tax rate, as follows:

\[
Y_{pot}(\text{e}^{\delta t^*\ln r^*}) = \frac{A}{1 - (1 - t)b + \mu k / \delta}. \tag{5}
\]

Equation (5) is a condition of Laffer-Keynesian equilibrium. It shows that for macroeconomic equilibrium to exist, all else being equal (for the given values of autonomous spending, the real cash balance of money, and potential output), the tax
rate \( t \) must be such that it satisfies equation (5). We will call this \( t \) the \textit{equilibrium average tax rate}.\footnote{The existence of equilibrium \( t \) is graphically illustrated in Figure 3, where \( Y^s \) represents the aggregate supply curve; and \( Y^d \), the aggregate demand curve. In this case, Figure 3a corresponds to the case when in \( Y^d(t) \) the marginal propensity for government purchases \( g \) is less than the marginal propensity to consume \( b \); Figure 3b, to the case when \( g > b \); and Figure 3c, to the case when \( g = b \).\footnote{As we see, with different relationships of \( b \) and \( g \) for given values of autonomous spending \( A_o \), the real cash balance \((M/P)\), and potential output \( Y_{pot} \), three cases can take place:}

1. No equilibrium average tax rate exists (the curves \( Y^s \) and \( Y^d \) do not intersect).
   It is clear that this happens when autonomous spending \( A_o \), which is part of \( A \), is so large that potential output \( Y_{pot} \) is not enough to satisfy it.

2. There are two equilibrium values of the average tax rate, \( t_1 \) and \( t_2 \) (the curves \( Y^s \) and \( Y^d \) intersect at two points), the role and values of which are determined by
the relationship between the marginal propensity of households to consume \( b \) and the marginal propensity for government purchases \( g \). When \( b > g \) (Figure 3a), of the two equilibrium values of the average tax rate the lower one \( t_1 \) is preferable, since it provides for greater aggregate spending; for the same reason, in the case of \( b < g \) (Figure 3b), the higher tax rate \( t_2 \) is preferable; and finally in the case of \( b = g \) (Figure 3c), both rates \( t_1 \) and \( t_2 \) provide the same result from the point of view of production, employment, and aggregate spending.

3. There is a single equilibrium tax rate (curves \( Y^S \) and \( Y^D \) touch each other at only one point). In the case of Figures 3a and 3b, this rate cannot be optimal.

Henceforth, for the sake of simplicity, of the three different relationships of \( b \) and \( g \) we will consider only one. In particular, we will assume that \( b > g \). We pointed out above that in this situation aggregate demand \( Y^D(t) \) is a decreasing function in relation to the average tax rate and this fully conforms to Keynesian theory. Here we note that the results for the case considered here are the same as those obtained in the other two cases.

**Versions of disruption and restoration of equilibrium**

To clarify how equilibrium is established in conditions of the given model, we turn to equation (5). Since for this equation the cases of nonexistence of a solution or the existence of only one solution (the first and third of the cases indicated above) are highly unlikely, we will assume that, for the given values of \( A \), \( Y_{pot} \), and \( \delta \), in relation to \( t \) (5) has two solutions: \( t_1 \) and \( t_2 \). Consequently, for curves \( Y^S \) and \( Y^D \) given in Figure 4, macroeconomic equilibrium can be found at one of the points \( F \) and \( E \). For clarity, we will assume that the starting point of economic equilibrium is \( F \), which corresponds to the average tax rate \( t_1 \).

Suppose that, for certain reasons, the initial equilibrium is disrupted. As follows from (5), the reason for this could be:

- a change in aggregate demand due to a number of circumstances, including an increase or decrease in one or more of the elements of planned autonomous spending or the real cash balance;
- a purposeful change in the tax rate by the government;
- a change in aggregate supply due to an increase or decrease in potential output \( Y_{pot} \).

We will consider each case separately.

**Restoration of equilibrium in the case of a change in aggregate demand**

We begin with the case when, all else being equal, aggregate demand undergoes a change. Specifically, we assume that it has increased thanks to an increase in some element of autonomous aggregate spending from \( A \). This change will cause
a shift of the aggregate demand curve $Y^D$ to a new position $Y_{1}^D$ (see Figure 4a). In the new situation that is created, as long as the tax rate is at level $t_1$, the amount of increased aggregate demand is determined by point $F_1$ and is greater than the amount of aggregate supply, which, for its part, is determined by point $F$. Depending on what actions the government takes, restoration of equilibrium or transition to a new equilibrium can be accomplished in two ways.

One way to establish equilibrium is to step up government activity, in particular, for it to improve tax administration and, by applying appropriate laws, to increase the value of $t$ from $t_1$ to $t_1'$, in parallel with the growth of autonomous spending. Such a measure will have an effect on the amount of aggregate demand as well as aggregate supply: increasing the tax rate will reduce the amount of aggregate demand (on the aggregate demand curve $Y^D$ it moves from point $F_1$ to $F_2$) and, according to the Laffer theory, increase the amount of aggregate supply (on the aggregate supply curve $Y^S$ it moves from point $F$ to $F_2$). As a result of these changes, a new equilibrium corresponding to the increased output is established at point $F_2$.

If the initial point of the economy’s equilibrium was not at point $F$, but at $E$, the situation would be different. According to Figure 4a, the latter point (i.e., $E$) is on the descending part of the aggregate supply curve, where the negative effects of taxes play a dominant role. Therefore, if in the hypothetically created situation the government lowers the value of $t$ from $t_2$ to $t_2'$, then the economy will be able to make the transition to a new equilibrium $E_2$ and ensure that the increased aggregate demand is properly satisfied.

The scenario considered above for restoration of macroeconomic equilibrium is interesting from the point of view that the government, by regulating the tax rate, makes it possible to satisfy increased aggregate demand so that the price level does not change. However, if the government takes a neutral position and refrains
from regulating the tax rate, then other mechanisms go to work, one of the most important of which is the market mechanism of regulation by prices.

From the standard model of aggregate demand and aggregate supply, which is analyzed in coordinates of the plane of price level and total output, it follows that when the economy is in a state of less than full employment and excess demand occurs, then, all else being equal, the price level rises. In the general case, this causes a decrease in aggregate demand and an increase in aggregate supply, so that equilibrium is established between them. Naturally, this mechanism also operates, with a certain specific nature, in the model (2)–(5) that we are examining. Its specific nature in model (2)–(5) is that in conditions of a fixed average tax rate an increase in the price level has an effect on the location of the aggregate demand and aggregate supply curves and causes them to shift. The form and direction of this movement for the case when the initial macroeconomic equilibrium is at point $F$ are shown in Figure 4b. Here, curves $Y_D^0$ and $Y_D^1$ express aggregate demand before the rise of the price level. In this case, $Y_D^0$ is the initial curve, and $Y_D^1$ is the curve that occurred after the increase in autonomous spending. Curve $Y_S$ corresponds to the supply that existed before the change in the price level.

The aggregate demand curve, which was lowered as a result of the rise of the price level, will take position $Y_D^2$, and the curve for the increased aggregate supply will take position $Y_S^1$. Consequently, the new equilibrium established as a result of the change in the price level is determined by point $F_2$.

We point out the circumstance that, in conditions of a fixed tax rate, a change in the price level caused by a change in autonomous spending and, accordingly, aggregate demand, has an effect on parameter $\delta$ and on the optimal tax rate determined by this parameter. When the initial equilibrium point is on the ascending part of aggregate supply (as it is in the case considered above), that is, when the tax rate $t_1$ corresponding to equilibrium is less than its optimal value $t^*$, the growth of aggregate demand and the price level are accompanied by a process of decrease of parameter $\delta$ and movement of the optimal tax rate to the left. In this case, if the process of growth of aggregate demand continues and, consequently, the price level also continues to rise, a time will come when the optimal tax rate becomes equal to the given fixed value of $t_1$ corresponding to equilibrium.

The opposite situation takes place when the initial equilibrium point is on the descending part of curve $Y_S$ and the tax rate corresponding to equilibrium is greater than its optimal value $t^*$. In these conditions, with a fixed value of the tax rate, growth of aggregate demand and, accordingly, the price level causes growth in parameter $\delta$, a shift of the aggregate supply curve to the left, and movement of the optimal tax rate toward the rate corresponding to equilibrium. In this case, if aggregate demand increases to a certain level, then the tax rate corresponding to the initial equilibrium will become the optimal rate.

The main conclusion from analyzing the version of restoration of equilibrium considered above is that with a change in parameter $\delta$ caused by a change in the price level, the equilibrium and optimal values of the tax rate come closer together.
We consider the case when the reason for disruption of equilibrium $F$ is a purposeful change in the tax rate $t_1$ by the government. As in the previous case, we will assume that the average tax rate $t_1$ (see Figure 5a), which is less than the fiscal Laffer point of the first kind $t^*$, corresponds to the initial equilibrium. Suppose that the government has decided to switch to optimal taxation $t^*$ and has raised the average tax rate accordingly. All else being equal, according to the Laffer theory, because of the dominance of positive effects, transition to the new tax regime $t^*$ will cause a growth in aggregate supply, which will be reflected accordingly on curve $Y^S$: there will be a transition from point $F$ to point $F_2$. At the same time, the increased tax rate will have an effect on aggregate demand, and movement from point $F$ to point $F_1$ will begin on curve $Y^D$. Consequently, as we can see from Figure 5a, an increase in the tax rate in some period will lead to the appearance of excess supply. In the situation that has been created, there are two possible ways of restoring equilibrium.

In one of them, in parallel with increased taxes, the government should promote growth of aggregate demand, for example, by making additional autonomous purchases or conducting an expansionary monetary policy. This will be a logical continuation of the actions that it has begun. If the government can increase the aggregate $A$ (which consists of elements of autonomous spending and the real cash balance) to the level $A = Y_{pot}(1 - b) + r^n(b - g) + \mu k/h$ in this way and the aggregate demand curve is shifted to position $Y^D_1$, then equilibrium is restored, with output reaching its potential value, so that the price level will not change (Figure 5a).

From the version of establishing equilibrium considered here, it follows that just putting the optimal tax rate into effect cannot ensure employment growth and initiate
the transition to equilibrium corresponding to potential output. In conditions of a Laffer-Keynesian synthesis, along with the tax regime, aggregate demand plays a significant role in achieving increased economic activity and full employment.

And the second way of restoring equilibrium convinces us of this. In fact, suppose that the government has limited itself to just increasing the tax rate and does not take any additional measures to stimulate aggregate demand. Then the task of overcoming the imbalance that has been created is shouldered by the economy’s main market mechanism; the price regulation mechanism. The point is that excess aggregate supply stimulates a reduction in the price level. In this process, aggregate demand begins to grow, and its curve shifts upward, from position $Y^D$ to position $Y^D_1$, while aggregate supply decreases, and its curve shifts from position $Y^S$ to the right, to position $Y^S_1$ (Figure 5b). Ultimately, a new equilibrium will appear at point $F_2$ instead of $F$, and it will correspond to less than the potential output. Moreover, rate $t^*$ will lose the function of optimality, and in the situation that has been created the role of the optimal tax rate will be assigned to $t^*_1$. Consequently, the government’s attempt to create full employment in the economy exclusively by adjusting the tax rate will not be successful if it is not assisted by appropriate aggregate demand.

**Restoration of equilibrium in the case of a change in aggregate supply**

Another cause of disruption of equilibrium could be a change in factors determining the level of potential output, for example, an increase or decrease in the existing amount of labor or capital, or deterioration or improvement in the level of technology. In all of these cases, $Y_{pot}$ which is part of the aggregate supply function, undergoes a change and, because of this, aggregate supply also changes. Geometrically, this circumstance is expressed in a shift of aggregate supply curve $Y^S$, except that, in contrast to the movements considered previously, in this case $Y^S$ shifts upward if $Y_{pot}$ increases, and downward if $Y_{pot}$ decreases.

Suppose that, in conditions of initial equilibrium $F$, the value of potential output was $Y_{pot}$, and the aggregate supply corresponding to it was described by curve $Y^S$ (Figure 6). Say that the value of $Y_{pot}$ has increased to $Y^1_{pot}$ because of an improvement in technology. All else being equal, this will cause a shift of the aggregate supply curve to position $Y^S_1$ and a temporary disruption of equilibrium: for the existing tax rate $t_1$, output will reach point $F_1$, provided that aggregate demand is determined by point $F$. A situation of excess aggregate supply is created, and there are three ways to bring it back into equilibrium:

1. To reduce the amount of aggregate supply and stimulate aggregate demand, the government should lower the existing average tax rate in the case of Figure 6a, and increase it to point $t_2$ in the case of Figure 6b. As a result of these actions, a new equilibrium is created at point $F_3$, which corresponds to higher
2. In conditions of the existing tax rate, by means of fiscal and monetary policy, the government should encourage an increase in aggregate demand to a level such that curve $Y^D$ shifts to point $F_1$. If this happens, then the new equilibrium will be at point $F_1$. If the government does not intervene in the process of creating equilibrium, then, because of the excess demand, the mechanism of self-regulation will take effect, and the price level will decline. On the one hand, this will increase aggregate demand, and the aggregate demand curve will shift to position $Y^D_1$; on the other hand, it will decrease aggregate supply and shift the aggregate supply curve from position $Y^S_1$ to $Y^S_2$. In the end, a new equilibrium position will be created at point $F_2$, at which the level of output and employment are higher than at the initial equilibrium point $F$.

We will call attention to the circumstance that, even in this case, a change in the price level causes a shift of the aggregate supply curve, and this is once again followed by a change in the optimal tax rate, which shifts from point $t^*$ to $t_1^*$. Hence the conclusion can be drawn that when the government keeps the average tax rate in a stable position, then each new equilibrium price level has its own optimal tax rate.

The Laffer curve

Analysis of model (2)–(5) of Laffer-Keynesian equilibrium has shown that when the tax rate is fixed, changes that occur in the economy have an effect on the aggregate
supply function (4) and cause a change in parameter $\delta$. This means that each given value of the equilibrium tax rate can correspond to a whole set of aggregate supply functions and, accordingly, supply curves that differ from each other in the value of parameter $\delta$ (or in the value of the optimal tax rate $t^*$, which is the same thing). Because of this, aggregate supply $Y^S$ can be seen as a function of two variables, $t$ and $\delta$:

$$Y^S = Y^S(t, \delta).$$

In this notation, all else being equal, $t$ reflects the weight of the tax burden. As for $\delta$, to explain its content we have to take into account the circumstance that the aggregate supply model (4) is, in essence, a behavioral model expressing the result of economic agents’ expected response in conditions of a tax burden of one weight or another. Naturally, in various situations existing in the economy, this response may not be the same for the same tax burden, the more so as the behavior and decision making of most economic agents is based on rationalism. And the latter, as we know, implies that economic agents optimize their behavior not just once, but over and over, continuously, taking each new circumstance into account. Consequently, parameter $\delta$ quantitatively expresses the result of the circumstances that may have an effect on the nature of the relationship that exists between aggregate supply and the tax burden.

Like the aggregate supply function, the version of the budget revenues function $T^S(t)$ determined on the basis of the supply function, that is, the Laffer function, should be seen as a function of two variables. In the conditions of model (2)–(5) under consideration here, it has the following form:

$$T^S(t) = tY^S(t) = Y^p(t) = Y^p(t) \left( -e^{\delta t} + \ln t \right),$$

where $T^S(t)$ is the amount of the budget’s tax revenues with a tax base $Y^S(t)$. Consequently, we can write:

$$T^S(t) = tY^S(t, \delta) = T^S(t, \delta).$$

It can be shown that tax rate $t^*$ corresponding to the maximum of the Laffer function (6), which is also called the Laffer point of the second kind, is determined as follows:

$$t^* = \exp\left( -\frac{1}{1+\delta} \right).$$

From this expression, it follows that with a change in $\delta$, the value of $t^*$ also changes, and, like the supply curves, a set, or family, of Laffer curves is obtained. However, in contrast to the aggregate supply curve, which has the same value of the maximum $Y^p$ for all $\delta$, the maximum value of the Laffer curve (6) is different for various $\delta$. The point is that for function (6), the following condition takes place:

$$\max_t T^S(t, \delta) = \frac{\delta}{1+\delta} Y^p.$$
Therefore, with a change in \( \delta \), the value of the tax rate \( t^* \) corresponding to the maximum of the Laffer curve changes, as does the maximum amount of tax revenues \( T^*(t^*, \delta) \) that can be obtained with that tax rate. In particular, \( T^*(t^*, \delta) \) increases with an increase in \( \delta \) and decreases with a decrease. Consequently, a shift of \( t^* \) to the right on the tax rate axis is accompanied by an increase in \( T^*(t^*, \delta) \).

Figure 7 shows aggregate supply curves and the budget revenues corresponding to them for different values of \( \delta \). In both parts of the figure (7a and 7b), it is understood that the relationship \( \delta_1 < \delta < \delta_2 \) takes place.
Suppose that, in a state of macroeconomic equilibrium, the aggregate supply curve corresponding to the given rate $t$ is $Y^S(t, \delta)$, and the Laffer curve is $T^S(t, \delta)$ (see Figure 7). Say that, in these conditions, aggregate demand has increased, and the government has decided to keep the tax rate at the existing level $t$. We already know that when the initial macroeconomic equilibrium is on the ascending part of the supply curve $Y^S(t, \delta)$, then in the newly created situation, new aggregate supply $Y^S(t, \delta_1)$ and Laffer functions and curves $T^S(t, \delta_1)$ will be formed, and the $\delta_1$ corresponding to these curves is less than $\delta_2$. If we assume that, in the same situation, the initial macroeconomic equilibrium is on the descending part of the supply curve $Y^S(t, \delta)$, then the new aggregate supply and Laffer curves and functions take the form determined by $Y^S(t, \delta_2)$ and $T^S(t, \delta_2)$, respectively. It is clear that opposite changes will occur in the case of a decrease in aggregate demand: the aggregate supply and Laffer curves will shift to positions $Y^S(t, \delta_2)$ and $T^S(t, \delta_2)$, respectively, if the initial macroeconomic equilibrium is on the ascending part of the initial equilibrium, and to the positions $Y^S(t, \delta_1)$ and $T^S(t, \delta_1)$, if it is on the descending part.

From the analysis that has been done, it follows that the Laffer curve is not a stable construct and can change depending on the situation that has been created in the economy, especially as a result of a change in the price level, which also implies a change in $t^*$. In such conditions, the widespread opinion among proponents of the theory that it is desirable to somehow determine the value of the tax rate $t^*$ that provides maximum tax revenues to the budget, which is the basis for developing economic policy and improving the existing tax regime, loses its attractiveness, since, because of the changes occurring in the economy, it will be necessary to change the established rate and in the final analysis this will produce an undesirable result.

Notes

3. To be more precise, $\tilde{G}^*_0$ includes not the full amount of government purchases (as is customary in traditional Keynesian models, but only the part of it that does not depend on tax revenues to the national budget and is determined exogenously.
5. Ibid., p. 131.
6. It should be noted that (2) differs from the well-known version of the Keynesian model of aggregate demand (from the IS–LM model) in the rule that reflects taxes and determines the aggregate $A$ of autonomous elements.
8. From the economic content of the average tax rate, it follows that $0 \leq t \leq 1$. These values also establish the area of determination of the function $Y^S(t)$. 
9. A detailed explanation of the reasons why aggregate demand is increasing in relation to the tax rate when \( b < g \) is given in Ananiashvili and Papava, “Rol’ srednei nalogovoi stavki v keinsianskoi modeli sovkupnogo sprosa.”


12. It is assumed that an increase in the tax rate is accompanied by positive and negative effects. For a value of the average tax rate in the range from 0 to \( t^* \), the sum of the positive effects exceeds the sum of the negative ones, and therefore movement on the ascending part of the aggregate supply curve corresponds to an increase in the tax rate in this interval. The opposite relationship of positive and negative effects occurs for subsequent values of the average tax rate, and so a decrease in aggregate supply corresponds to an increase in the tax rate in the section \((t^*, 1]\). For a detailed explanation of this question, see ibid.

13. Recall that in the aggregate supply model \((4)\), \( t^* \) and parameter \( \delta \) are interrelated as 
\[
t^* = e^{-1/\delta}.
\]

14. The case when the economy is in a state of full employment and tax rate \( t \) coincides with optimal value \( t^* \) is an exception. In this case, an increase in the level of prices does not affect either the maximum of the function \( Y^D(t) \), the parameter \( \delta \), or the tax rate \( t^* \) corresponding to it. Consequently, in this case the aggregate supply curve stays where it is.

15. It is easy to notice that when the aggregate supply curve shifts to the right, the value of the optimal tax rate decreases, and when the curve shifts to the left, it increases. Taking this into account, from the expression determining the optimal tax rate \((t^* = e^{-1/\delta})\), we will establish that a decrease in the value of parameter \( \delta \) corresponds to a shift of the curve \( Y^S(t) \) to the left; and an increase in \( \delta \), to a shift to the right.

16. Based on the specifics of equation \((5)\), in the general case it is not possible to derive an analytical expression for equilibrium \( t \), but it is no problem to find it by approximate calculation methods.

17. Recall that in the suggested version of the Keynesian model of aggregate demand, the relationship of the \( b \) and \( g \) determines the nature of the dependence of aggregate demand on the average tax rate. In particular, \( Y^D(t) \) decreases in relation to \( t \) when \( b > g \), increases when \( b < g \), and is indifferent when \( b = g \).

18. We are dealing with precisely such a case.

19. A change in the price level within the limits of model \((2)-(5)\) means a disruption of “all else being equal.” Therefore, on the coordinate plane of the tax rate and output a change in the price level has an effect on the location of the curves \( Y^D \) and \( Y^S \), while in the standard model of aggregate demand and aggregate supply, a change in the price level causes movement on the curves \( Y^D \) and \( Y^S \).

20. The point is that when \( P \) rises, all else being equal, \( A \) decreases, since the latter includes the element \( MP \), which depends on \( P \) (see equation \([3]\)).


22. This formula is directly derived from \((5)\) by substituting \( t^* \) into it and taking into account that \( f(t^*) = -e(t^*)^\delta \ln(t^*)^\delta = 1 \).

23. We pointed out above that in model \((2)-(5)\) a change in the price level affects the position of the aggregate demand and aggregate supply curves.

24. The distinctive feature of these two ways of establishing a new equilibrium is that the government, by conducting a well-thought-out economic policy, makes it possible to increase the level of employment and production so that the price level stays at the point where it was in conditions of the initial equilibrium.
25. As we pointed out above, among these circumstances a special role is played by a change in the price level, therefore, instead of $\delta$ we can use the notation $\delta = \delta(P)$, and the notation of the aggregate supply function $Y^s(t)$ can be replaced by $Y^s(t, \delta(P))$.


27. Ananiashvili and Papava, “Modeli otsenki vlianiia nalogov na rezul’tat’ taty ekonomicheskoi deiatel’nosti.”

28. It should be noted that there are now many articles and studies confirming or refuting the concept of the Laffer curve and problems of its practical realization (see, e.g., V. Papava, “Lafferov effect s posledeistviem,” Mirovaja ekonomika i mezhdunarodnye otosheniiia, 2001, no. 7; V. Papava, “On the Laffer Effect in Post-Communist Economies (On the Basis of the Observation of Russian Literature), Problems of Economic Transition, 2002, vol. 45, no. 7, pp. 63–81). Some of them deny the existence of this curve, and some confirm it a priori. But, except for the case of dividing the curves into long- and short-term periods (Dzh. M. B’iukenen [J.M. Buchanan] and D.R. Li [Lee], “Politika, vremia i krivaja Laffera” [Politics, Time, and the Laffer Curve], in M.K. Bunkina and A.M. Semenov, Ekonomicheskii chelovek: V pomoshch’ izuchaiushchim ekonomiki, psikhologii, menedzhment (Moscow: Delo, 2000), all of them unequivocally analyze a single curve, rather than a set of curves.


30. In the opinion of the well-known economist Robert Barro, constant changes in the tax rate (either increases or decreases) cause greater distortions and irrecoverable losses than a tax regime that has a fixed rate (see, e.g., Dzh. Saks [J. Sachs] and F.B. Larren [Larrain], Makroekonomika. Global’nyi podkhod [Macroeconomics in the Global Economy] [Moscow: Delo, 1996], p. 245).