Problems of Economic Transition, vol. 54, no. 12, April 2012, pp. 71–91.

ISSN 1061–1991/2012 $9.50 + 0.00.
DOI 10.2753/PET1061-1991541205

Iuri Ananiashvili and Vladimer Papava

Taxes, Production Technology, and Economic Growth

The article examines two different approaches to estimating the effect of the tax burden on the amount of total output and budget revenues. The first approach is based on a transformation model, in which the main role is played by a production function with variable elasticity. The second approach uses a behavioral model, with a specific version of an entropy function. Both models make it possible to determine the so-called fiscal points corresponding to the maximum production effect and the budget's maximum tax revenues. The conclusion is drawn that, of these points, only the points of the behavioral model correspond to the Laffer concept, since for points derived from the transformation model the amount of use of economic resources is exogenous, while for the points of the behavioral model this amount occurs endogenously. The results obtained are illustrated using existing data on the U.S. economy.

The fact that the modern state and society could not exist without taxes needs no special proof. At the same time, it is recognized that taxation has an effect on consumption and savings, investment, supply and demand, pricing, the scale of markets, and so on.[1] In the final analysis, all of this directly or indirectly affects the amount of production and budget revenues.

English translation © 2012 M.E. Sharpe, Inc., from the Russian text © 2010 "Obshchestvo i ekonomika" and the authors. "Nalogi, tekhnologiia proizvodstva i ekonomicheskii rost," *Obshchestvo i ekonomika*, 2011, no. 4–5, pp. 172–96. A publication of the International Association of the Academies of Sciences.

Iuri Ananiashvili is a Doctor of Economic Sciences and professor in the Department of Econometrics of Ivane Javakhishvili Tbilisi State University; e-mail: iuri_ananiashvili@yahoo.com. Vladimer Papava is a Doctor of Economic Sciences, professor, corresponding member of the National Academy of Sciences of Georgia, and chief research fellow at the Paata Gugushvili Institute of Economics; e-mail: papavavladimer@yahoo.com.

Translated by James E. Walker.

The tax burden can affect the amount of output and the budget's tax revenues in two different ways. On the one hand, it has an impact on production technology and the efficiency of resource use, and it influences output and budget revenues in this way. On the other hand, a change in the tax burden has an impact on the amount of use of economic resources and causes growth or contraction of production and budget revenues according to the change in the involvement of resources in production. Both of these effects can be analyzed and estimated based on mathematical economic models.

Two such models are presented in this article. In one model, the tax burden (average tax rate) is a factor determining the technology and efficiency of resource use, while in the other it is a factor determining the amount of resource use and the level of economic activity. Both of these models consider the values of total output and budget revenues as functions depending on the aggregated tax rate. If total output is designated as Y, and the budget's tax revenues as T, then we can write $Y = Y(t)$ and $T = T(t)$, where t is the aggregated (average) tax rate, which satisfies the condition $0 \leq t \leq 1$. In this case, it is understood that the functions $Y(t)$ and $T(t)$ are interrelated as $T(t) = tY(t)$. This relationship shows that the behavior of the budget revenues function is substantially determined by the behavior of $Y(t)$, Therefore, of these two functions, in the models to be considered below, more attention will be given to the total output function $Y(t)$.

Model for estimating the effect of the tax burden on production technology

At the theoretical level, it is rather complicated to clearly substantiate how the tax burden impacts the technological relationship that objectively exists between resource expenditures and the maximum amount of output in conditions of these expenditures. At the same time, it is absolutely logical to assume that in the same (equal) technological conditions (with the same amounts of labor and capital), a different level of the tax burden will result in a different amount of gross domestic product (GDP).[2] The point is that, in the case of taxation, activity and individual types of products for which taxes are a considerable burden are replaced by activity and products that are less problematic from the standpoint of taxes. The return on certain alternatives for resource use declines, the return on other alternatives grows in parallel, and a new structure of production and consumption is formed, which is accompanied by redistribution of initial resources between forms of activity and a change in efficiency of the production process.

In the context of production technology, to quantitatively estimate the dependence of output on the amount of the tax burden, we can use expansions of the macroeconomic production function in which the role of the average tax rate is distinguished in some form. Such an expansion is possible in two basic directions. In one direction, taxes should be seen as a component of production technology.

If we take the Cobb–Douglas production function, for example, as the basic one, then, in this case, possible versions of its expansion by means of taxes will be:

$$Y(t) = \gamma D t^{\lambda} K^{\alpha} N^{\beta};\ \ Y(t) = \gamma D e^{\lambda t} K^{\alpha} N^{\beta},$$

where $Y(t)$ is total output; K is the cost of the capital used; N is the amount of labor used; t is the aggregated (average) tax rate (the ratio of the budget's total tax revenues to GDP); e is the base of the natural logarithm (Neperian number); D is a trend operator (a function of which the argument is time); α is the capital elasticity coefficient of output; β is the labor elasticity coefficient of output; and γ and λ are parameters the statistical estimation of which, together with other parameters, is done based on time series of the variables $Y(t)$, K, N, and t.

In the second way of expanding the production function, taxes are seen not as components of technology, but as factors that act on the efficiency of technology, or rather, on the efficiency of the resources used in technology: labor and capital. We will analyze one version of such an expansion below. Suggested by Evgeny Balatsky,[3] it is a production function with variable elasticity, in the following form:

$$Y(t) = \gamma D K^{\alpha(t)} N^{\beta(t)}, \tag{1}$$

where $\alpha(t)$ and $\beta(t)$ are the capital and labor elasticity coefficients of output, the values of which depend on the average tax rate t.

It should be noted that function (1) and the budget revenues function corresponding to it:

$$T(t) = tY(t) = t\gamma D K^{\alpha(t)} N^{\beta(t)}, \tag{2}$$

(or, on the whole, a model such as [1]–[2]) was developed by Balatsky for a broader purpose than that in connection with which we are talking about it in this case: for substantiating the macroeconomic concept of the Laffer curve and estimating the effect of fiscal policy on the level of business activity in a country with reasonable reliability.[4] In spite of this, we believe that in modeling the relationship of the average tax rate and output, even with the version of the expanded production function (1), it is only partially possible to reflect the essence of the Laffer concept. The point is that the underlying essence of the Laffer theory—that is, its philosophy—consists in the idea that an increase or decrease in the tax burden, by creating a negative or positive system of stimuli, fosters a decline or growth in economic activity, which is primarily expressed in a change in the amount of use of resources, rather than in an increase or decrease in the efficiency of their use. Consequently, to characterize the main aspect of the Laffer theory requires a model that is based on a behavioral equation and can reflect the positive and negative stimuli created by taxes, rather than a model based on the transformation equation (1), which for the most part is used to characterize production technology.[5]

Although (1) is not a behavioral equation that can reflect the positive and negative stimuli created by taxes, model (1)–(2), on the whole, is a tool with broad theoretical possibilities. There are two reasons for this. The first involves the specific nature of the production function itself. As we know, the Cobb–Douglas production function, on which this model is based, can be used to calculate and analyze many technical economic characteristics.[6] The second reason involves consideration of the institutional factor. In particular, including the tax rate in a model of the production function and adopting the hypothesis that the tax burden has an effect on production technology and the efficiency of resource use (in our view, the model is based precisely on such a hypothesis) enables us to analyze the technical economic characteristics obtained from a typical production function from a new angle. And this angle is expressed in the fact that all of the basic parameters to be analyzed, which are obtained using a total output function (1) and a tax revenues function (2) corresponding to it, are explicitly or implicitly related to the tax burden.

It is easy to see that in model (1)–(2), the impact of the tax burden on an economic system and its characteristics is realized through the capital and labor elasticity coefficients of output $\alpha(t)$ and $\beta(t)$, which, according to the adopted hypothesis, are functions that depend on the average tax rate t. Therefore, $\alpha(t)$ characterizes the percent change in output with a 1 percent change in the amount of capital used in conditions of taxation at the rate t. The content of $\beta(t)$ is analogous, only in this case the percent change in output is considered in relation to a 1 percent change in the amount of labor used in conditions of the tax rate t.

The choice of a specific type of function $\alpha(t)$ or $\beta(t)$ should be based on general theoretical considerations, the specific nature of existing statistical data, and the possibility of adequately interpreting the results of the model that is being estimated. If we use theoretical considerations and take into account that, in addition to production-technology aspects, the model should be applicable to the analysis of certain fiscal problems, then the following functions can be deemed acceptable:

$$\alpha(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2, \tag{3}$$

$$\beta(t) = \beta_0 + \beta_1 t + \beta_2 t^2, \tag{4}$$

where α_j and β_j, $j = 0, 1, 2$ are the parameters to be estimated. In this case, it is desirable that at least one of the parameters α_2 and β_2 be nonzero (in other words, it is desirable that at least one of the functions $\alpha(t)$ and $\beta(t)$ be quadratic).[7]

The advisability of selecting quadratic functions of the elasticity coefficients $\alpha(t)$ and $\beta(t)$ is primarily due to the fact that in the conditions of such functions there can be a maximum point for (1) and (2) in relation to t. If these points are in the range of permissible values of the tax rate—that is, in the interval [0, 1]—then they can be called *Balatsky's fiscal points of the first and second kind,* since Balatsky was the first to explicitly examine these two points.[8] Here it should be noted that in his articles, Balatsky called these values of the tax rate corresponding to the maximums of output and tax revenues that are derived based on model (1)–(4) Laffer points of the first and second kind. However, in our view, this name is not entirely correct, since model (1)–(4) does not completely satisfy the postulates of the Laffer theory[9] and, most important, the basic equation (1) in model (1)–(4) is not a behavioral equation.

We will designate the value of the tax rate corresponding to the maximum of (1) as t^Y, and the value of the tax rate corresponding to the maximum of (2) as t^T. Then to determine t^Y we should study the equation $\partial \ln Y/\partial t = 0$, and to derive t^T, the equation $\partial \ln T/\partial t = 0$. After performing appropriate transformations in these equations, we find that Balatsky's fiscal point of the first kind t^Y—that is, the point for which output is maximum—is determined as follows:

$$t^Y = -\frac{\alpha_1 \ln K + \beta_1 \ln N}{2(\alpha_2 \ln K + \beta_2 \ln N)}. \tag{5}$$

Balatsky's fiscal point of the second kind t^T corresponds to the equation:

$$t^T = \frac{1}{2}\left(t^Y \pm \sqrt{(t^Y)^2 - \frac{2}{\alpha_2 \ln K + \beta_2 \ln N}} \right). \tag{6}$$

Equations (5) and (6) are somewhat simplified for cases when one of the functions $\alpha(t)$ and $\beta(t)$ is linear and the other is quadratic. If we assume that $\alpha(t) = \alpha_0 + \alpha_1 t$, then (5) and (6) will take the following forms:

$$t^Y = -\frac{\alpha_1 \ln K + \beta_1 \ln N}{2\beta_2 \ln N}; \qquad t^T = \frac{1}{2}\left(t^Y \pm \sqrt{(t^Y)^2 - \frac{2}{\beta_2 \ln N}} \right).$$

And in the case when the labor elasticity coefficient of output $\beta(t)$ is linearly determined (i.e., $\beta(t) = \beta_0 + \beta_1 t$), we get

$$t^Y = -\frac{\alpha_1 \ln K + \beta_1 \ln N}{2\alpha_2 \ln K}; \qquad t^T = \frac{1}{2}\left(t^Y \pm \sqrt{(t^Y)^2 - \frac{2}{\alpha_2 \ln K}} \right).$$

As we see, the values of points t^Y and t^T depend on the relationship of use of capital and labor. Depending on the sign and value of coefficients α_j and β_j, $j = 1, 2$, in a specific situation, for a given value of capital provision, permissible t^Y and t^T (in the interval [0, 1]) may or may not exist. In this case, if for a given K/N (level of capital provision) there is a permissible value $t^Y \in [0, 1]$ of Balatsky's fiscal point of the first kind, it is the only one. As for Balatsky's fiscal point of the second kind t^T, it is easy to see that its behavior depends significantly on the behavior of t^Y. At the same time, t^T is characterized by a certain specific nature due to the existence of a radicand in (6). Depending on the sign and value of the expression $\alpha_2 \ln K + \beta_2 \ln N$ for given N and K, there may not be any fiscal point t^T in the interval [0, 1], there may be one such point, or there may be two. From (6), it follows that:

(a) When $\alpha_2 \ln K + \beta_2 \ln N < 0$, then for a given t^Y ($t^Y \in [0, 1]$) there can be only one t^T. In this case, it will satisfy the condition $t^Y < t^T$, which means that the maximum production effect will be achieved in conditions of a tax rate lower than the one at which the maximum budget revenues are obtained.

(b) When $\alpha_2 \ln K + \beta_2 \ln N > 0$, then for a given t^Y ($t^Y \in [0, 1]$), either there is no real t^T, or there are two values of it, which are different from each other. In the latter case, both of them are in the interval [0, 1], and their values are less than the fiscal point of the first kind t^Y: $t^T < t^Y$. It is clear that, of these two values of t^T, the one that should be considered as the fiscal point of the second kind is the global maximum point—that is, the t^T that corresponds to the highest tax revenues.

One more circumstance should be pointed out. Infinite growth of K in conditions of a given N, or vice versa, infinite growth of N in conditions of a given K, causes infinite growth in the modulus of the expression $\alpha_2 \ln K + \beta_2 \ln N$. As follows from (6), in this case $t^T \to t^Y$. Consequently, according to model (1)–(4), if the amount of use of some production factor grows infinitely, then the difference between Balatsky's fiscal points of the first and second kind gradually disappears.

Based on model (1)–(4), along with the fiscal characteristics t^Y and t^T, significant technological characteristics are also obtained. Among them, first of all, we should note the marginal product of capital $MPK(t)$ and the marginal product of labor $MPN(t)$:

$$MPK(t) = \frac{\partial Y(t)}{\partial K} = \alpha(t)\frac{Y(t)}{K}, \tag{7}$$

$$MPN(t) = \frac{\partial Y(t)}{\partial N} = \beta(t)\frac{Y(t)}{N}. \tag{8}$$

These relationships clearly show that in model (1)–(4), all else being equal, the marginal efficiency of each factor depends not only on the amount of its use (as is customary in comparatively simple production functions) but also on the existing average tax rate t. In a normal economy, in conditions of a moderate tax burden, based on their economic content the values of the marginal products of capital and labor $MPK(t)$ and $MPN(t)$ should be nonnegative. The point is that (1), as a production function, is essentially a transformation model, and in conditions of a moderate tax burden it should reflect a well-known technological pattern: if the amount of use of a resource in production increases, all else being equal, the total output, if it does not increase, at least should not decrease. On the other hand, since we are examining the tax burden as a factor affecting the efficiency of technology, it is entirely possible that with a very high average tax rate the marginal product of a factor could turn from positive to negative.

From equations (7) and (8), it follows that the condition of simultaneous nonnegativity of $MPK(t)$ and $MPN(t)$ for model (1)–(4) will be observed only when

Table 1

Ranges of Nonnegativity of Elasticity Coefficients $\alpha(t)$ and $\beta(t)$

	$\alpha(t) \geq 0$	$\beta(t) \geq 0$
Russia (1989–2000)	$0.7451 \leq t \leq 1$	$0 \leq t \leq 0.7425$
Sweden (1980–1994)	$0 \leq t \leq 0.5785$	$0.6145 \leq t \leq 1$
Great Britain (1983–1999)	$0 \leq t \leq 0.4756$	$0 \leq t \leq 0.3501$
United States (1986–2000)	$0 \leq t \leq 0.3266$	$0.26 \leq t \leq 1$

the values of $\alpha(t)$ and $\beta(t)$ in the range of determination of the average tax rate [0, 1] satisfy the following system of inequalities:

$$\alpha(t) \geq 0,\ \ \beta(t) \geq 0,\ \ 0 \leq t \leq 1. \tag{9}$$

However, as practice shows, in the case of quadratic functions this condition may not always be fulfilled. To illustrate this, we turn to Table 1, which gives results obtained by Balatsky based on econometric versions of model (1)–(4) for the economies of Russia, Sweden, Great Britain, and the United States.[10]

According to these results, for the Russian and Swedish economies, the set of solutions of the system of inequalities (9) is empty, if we disregard a zero tax rate.[11] This indicates that, according to model (1)–(4), for these countries there is no permissible nonzero value of the tax rate for which the values of the marginal products of capital and labor will be nonnegative at the same time. In the other two countries we have a comparatively better situation. For the British economy, the set of nonzero solutions of the system (9) is the inequality $0 \leq t \leq 0.35$, and for the U.S. economy it is $0.26 \leq t \leq 0.33$. As we see, according to model (1)–(4), in the British economy simultaneous fulfillment of the conditions $MPK(t) \geq 0$ and $MPN(t) \geq 0$ is possible in a fairly large range of values of the tax burden. It is strange, however, that in the years for which model (1)–(4) was estimated (1983–99) the tax burden that actually existed in Great Britain was outside of this range, with the exception of certain years (in particular, 1992–94) and, for the most part, provided for positive average annual values of $MPK(t)$ and $MPN(t)$.[12] It is true that the model's estimated results for the U.S. economy are free of such strange results, but even here not everything is as it should be. The problem is that, according to the econometric version of model (1)–(4) estimated for the United States, the condition of nonnegativity of $MPN(t)$ is $0.26 \leq t \leq 1$ (see Table 1). But in this interval, $MPN(t)$ is increasing in relation to t, and so is the labor elasticity of output $\beta(t)$, the specific form of which is $\beta(t) = 127.63t^2 - 33.18t$. It is hard to explain economically why inordinate growth of the tax burden should cause growth in the marginal product of labor.

Another example of inappropriate behavior of parameters obtained from model

(1)–(4) in relation to the tax burden is the scale efficiency of production ($\alpha(t) + \beta(t)$). This mathematically expresses the degree of homogeneity of function (1) and economically shows what happens with the average cost per unit of production with an increase in the scale of production. An increase in scale implies growth in both resources (factors) included in the model by a factor of $\rho(\rho > 1)$. If the tax burden in a given economic system has a significant effect on the production technology, it is not ruled out that for various values of t all three of the cases given below may occur:

(a) $\alpha(t) + \beta(t) = 1$—which means that in conditions of the given tax burden the level of efficiency does not depend on the scale of production. In this case, we say that in conditions of the given t there is a constant effect in relation to scale;
(b) $\alpha(t) + \beta(t) > 1$—for all values of the tax rate for which this inequality is valid, growth in the scale of production lowers the total average cost per unit of output, that is, in conditions of the given t an increasing effect of scale is at work;
(c) $\alpha(t) + \beta(t) < 1$—growth in the scale of production is characterized by decreasing efficiency for all t that are solutions of the given inequality.

We turn again to the econometric versions constructed by Balatsky for model (1)–(4),[13] and determine for what values of t the economies of Russia, Sweden, Great Britain, and the United States have a constant, increasing, or decreasing effect in relation to scale. Since the parameters of these versions were obtained for a model in which $\alpha(t) = \alpha_1 t + \alpha_2 t^2$ and $\beta(t) = \beta_1 t + \beta_2 t^2$, the degree of homogeneity ($\alpha(t) + \beta(t)$) of function (1) is determined in the following form:

$$\alpha(t) + \beta(t) = (\alpha_2 + \beta_2)t^2 + (\alpha_1 + \beta_1)t.$$

Taking into account the specific estimated values of the parameters α_j and β_j, $j = 1, 2$, we find that the form of the output effect (constant, increasing, or decreasing) in relation to scale is determined by the following conditions:

- for Russia (1989–2000): $-6.32t^2 + 4.68t\ (=, >, <)\ 1$;
- for Sweden (1980–94): $-1.71t^2 + 0.82t\ (=, >, <)\ 1$;
- for Great Britain (1983–99): $-105.18t^2 + 38.9t\ (=, >, <)\ 1$;
- for the United States (1986–2000): $81.76t^2 - 18.76t\ (=, >, <)\ 1$.

From these mathematical expressions, it follows that for the Russian and Swedish economies the form of the scale effect does not depend on the amount of the tax burden. The point is that however the value of t changes in the range that is permissible for it, $0 \leq t \leq 1$, a decreasing effect in relation to scale is maintained in both of these countries, since for any permissible value of t the inequality $-6.32t^2 + 4.68t < 1$ is fulfilled for Russia, and the inequality $-1.71t^2 + 0.82t < 1$ is fulfilled for Sweden.

We have a completely different situation for the British and U.S. economies: in these countries, the form of the effect in relation to scale depends significantly on the value of the tax rate. For example, in the British economy there is a constant effect of scale when the average tax rate is 0.0278 and 0.3421, a decreasing effect when $0 \leq t < 0.0278$ and $0.3421 < t \leq 1$, and, finally, an increasing effect when $0.278 < t < 0.3421$. And in the U.S. economy we have the following pattern: $t = 0.2682$ is a constant effect, $0 \leq t < 0.2682$ is a decreasing effect, and $0.2682 < t \leq 1$ is an increasing effect.

As we see, according to the econometric models constructed by Balatsky, the British and U.S. economies differ considerably from each other in the structure of distribution of taxes determining the form of their effect in relation to scale: the switch from one form of the scale effect to another occurs in conditions of a completely different tax burden. There is nothing unexpected in this, but it is strange that in the United States an inordinate increase in the tax rate is a condition for growth of the efficiency of scale. In particular, as the results given above show, the scale effect becomes increasing only when the tax burden in the U.S. economy exceeds approximately 27 percent[14] and continues to grow to the 100 percent level that is theoretically permissible for it. It is hard to find any valid explanation of this fact. Presumably it is a manifestation of imperfection of the model's specification or identification.

In spite of individual contradictions revealed in the process of analysis of econometric versions of model (1)–(4), there is no doubt that model estimation and analysis of the tax burden's effect on existing technological relationships between total production and the amount of capital and labor used in production is possible. The Cobb–Douglas function may not prove to be suitable for describing these relationships in all cases, even in the generalized form in which it is presented in model (1)–(4), and it may be necessary to use another, more complex production function.[15] But even when model (1)–(4) is satisfactory, from both the standpoint of formal statistical criteria and the standpoint of being able to interpret the results, it only partially reveals the role that taxes play in the economy. We have indicated above that the tax burden has an impact on production technology as well as on economic activity and the level of use of existing resources. We believe that this last circumstance is much more important from a macroeconomic point of view, and therefore primary attention should be given to it in modeling fiscal aspects and examining the role of taxes.

Model for estimating the effect of the tax burden on the amount of resource use

We can base the construction of this type of model on a generalized version of the concept of one of the representatives of Arthur Laffer's economic theory of supply, according to which the aggregated (average) tax rate has an impact on total output in approximately the same form as on the amount of the budget's tax revenues.[16] Postulates of this concept can be formulated in a formalized way as follows:

1. At the extreme points $t = 0$ and $t = 1$ of the range of determination of the aggregated (average) tax rate, the values of total output $Y(t)$ and budget revenues $T(t)$ are equal to zero, that is:

   $Y(0) = Y(1) = 0, T(0) = T(1) = 0.$

2. There are values $t^* \in [0, 1]$ and $t^{**} \in [0, 1]$ of the average tax rate t such that $Y(t)$ increases in the interval $[0, t^*)$ and decreases in the interval $(t^*, 1]$, and $T(t)$ increases in the interval $[0, t^{**})$ and decreases in the interval $(t^{**}, 1]$. In this case:

   $\max_{0 \le t \le 1} Y(t) = Y(t^*), \max_{0 \le t \le 1} T(t) = T(t^{**}).$

The average tax rate t^* at which output is maximum is called *the Laffer fiscal point of the first kind*, and t^{**} that produces the maximum budget revenues is called *the Laffer fiscal point of the second kind*.[17] It is clear that of the two points the more important one for the economy is the point of the first kind t^*. Therefore, we arbitrarily call t^* *the optimal tax rate*.

Determination of the fiscal points t^* and t^{**} can be one of the conditions fostering improvement of a country's economic policy. Two circumstances should be taken into account when constructing an appropriate model. The first one is that, in any economy, the total output depends on the amount and quality of existing economic resources (labor, capital, land, and production capabilities) and on the level of technology for using these resources. These factors determine the economy's production-technology capabilities, and if they are distributed in the best possible way and fully used we have the maximum output, which is also called *the potential output level*. The second circumstance is that no less a role in the economy is played by the institutional environment, creation of which is a function of the government. Depending on how ideal the institutional environment is, in conditions of the same production-technology capabilities, the amount of output will be different for any two economies or any two periods of time. In the case of the best, that is, ideal, institutional environment, the actual and potential outputs are equal to each other. However, as a rule, the actually existing institutional environment differs from its ideal version in most cases. Therefore, the actual level of the economy's total output is less than the potential level. Without question, an important role in creating the institutional environment is played by the tax system, along with a set of other factors. At the level of a model, we can set up a situation and assume that it is the tax system that is the main factor in creating the institutional environment that determines the behavior of economic agents. If we make such an assumption, then the total output function $Y(t)$ can be represented in the following form:

$$Y(t) = Y_{pot} f(t), \tag{10}$$

where Y_{pot} is the result expressing the economy's production-technology capabilities; and $f(t)$ is the function reflecting the institutional aspect.

From a formal point of view, Y_{pot} represents the maximum value of any macroeconomic production function in conditions of the optimal institutional environment. More specifically, Y_{pot} expresses the amount of potential output in conditions of the existing technology with full use of economic resources.

As for the function $f(t)$ in (10), it describes the overall effect of taxes on total output. It is a behavioral function that, based on its content, should have the following properties:

1. $f(t)$ is increasing in the interval $[0, t^*)$ and decreasing in the interval $(t^*, 1]$. In other words, from 0 to t^* an increase in the tax rate fosters an improvement of the institutional environment and growth in economic activity, while from t^* to 1 an increase in the tax rate leads to deterioration of the institutional environment and a decline in economic activity;
2. For the optimal tax rate, $f(t^*) = 1$. This very important property indicates that the average tax rate t^* makes it possible to create an institutional environment in which the technological aspects of production completely determine the efficiency of output. Consequently, with the optimal average tax rate, output is maximum, and (10) takes the form: $Y(t^*) = Y_{pot}$;
3. It is desirable for $f(t)$ to have one more property. In particular, in the absence of taxes, hat is, when $t = 0$, $f(0) = 0$, while if the profit that is made is completely confiscated in the form of taxes, that is, if $t = 1$, then $f(1) = 0$. However, it should be noted that $f(t)$ may not satisfy the third property, fully or partially. For example, for the case $t = 0$, $f(t)$ will be different from zero if we suppose that there are state-owned firms and the government performs economic functions based on income received in the form of dividends from their profits.

We give an example of a total output function corresponding to (10), in which $f(t)$ has the properties enumerated above. For this purpose, we use a modified version of the entropy function $(-t\ln t)$:[18]

$$f(t) = -et^{\delta}\ln t^{\delta}. \tag{11}$$

Then we have:

$$Y(t) = Y_{pot} f(t) = Y_{pot}(-et^{\delta}\ln t^{\delta}), \tag{12}$$

where δ is a statistically estimated potential parameter; and e is a Neperian number (base of the natural logarithm).

The budget revenues function corresponding to (12) has the following form:

$$T(t) = tY(t) = tY_{pot} f(t) = Y_{pot}(-et^{\delta+1}\ln t^{\delta}). \tag{13}$$

It can be shown that, in the conditions of model (12)–(13), the values of the Laffer fiscal points of the first and second kind, t^* and t^{**}, are determined as follows:

$$t^* = \exp\left(-1/\delta\right) = e^{-1/\delta}, \qquad t^{**} = \exp\left(-1/(\delta+1)\right) = e^{-1/(\delta+1)}. \tag{14}$$

In addition, the following conditions are valid:

$$\lim_{x\to 0} f(t) = 0,\ f(t^*) = 1,\ f(1) = 0.$$

Therefore, for the total output function (12) we have:

$$\lim_{t\to 0} Y(t) = 0\ \ Y(t^*) = Y_{pot},\ \ Y(1) = 0.$$

And for the budget revenues function (13):[19]

$$\lim_{t\to 0} T(t) = 0,\quad T(t^{**}) = \frac{\delta}{1+\delta} Y_{pot},\quad T(1) = 0.$$

As we see, in the conditions of model (12)–(13), the value of the fiscal characteristics t^* and t^{**} depend completely on parameter δ. To estimate the latter and, consequently, to identify model (12)–(13) we need observation data in relation to total output $Y(t)$, the tax rate t, and the potential output level Y_{pot}. The last of these, Y_{pot}, is not observable, that is, it is a latent quantity. Therefore, determining (estimating) its value requires developing a definite procedure, which is a separate problem.[20]

To solve the problem involving Y_{pot}, in model (12)–(14) we have to keep in mind that the potential output level Y_{pot}, in contrast to the actual level, is determined by the amount of economic resources that exist but are not used. If we take only two aggregated resources into account—labor and capital—then we can write

$$Y_{pot} = \varphi(\Phi, L), \tag{15}$$

where Φ is the existing amount of capital; L is the existing amount of labor; and φ is some function that can be estimated, which can arbitrarily be called the technological function of potential output. This function cannot be estimated in isolation, by examining the expression $Y_{pot} = \varphi(\Phi, L)$ only, since, as we pointed out above, we do not know the values of Y_{pot} in it. At the same time, if the value of Y_{pot} in the total output function (10) is replaced by the function $\varphi(\Phi, L)$ and the expression obtained

$$Y(t) = \varphi(\Phi, L)f(t) \tag{16}$$

is transformed into a regression equation, then, along with $f(t)$ we can also estimate $\varphi(\Phi, L)$.

To illustrate this, we turn to statistical data that exist for the U.S. economy and consider 1970–2008 the period to be analyzed.[21] To determine the specific econometric form of (16), we represent the potential output function (15) as follows:

$$Y_{pot(i)} = Ae^{\lambda i} L_i^{\mu} L_{i-1}^{\eta} Y_{i-1}^{\theta}\ , \tag{17}$$

where i is the time index; $Y_{pot(i)}$ is potential output in period i; A, λ, μ, η, and θ are parameters that can be statistically estimated; L_i and L_{i-1} are the amount of labor in periods i and $i - 1$, respectively; and Y_{i-1} is actual output in period $i - 1$.

Several circumstances determined the choice of such a structure for the potential output function. The first one involves overcoming the problem of autocorrelation. The lag variables (L_i and L_{i-1}) are included in the model mostly for this purpose, although taking these variables into account expands the context of economic analysis, since it becomes possible to reflect dynamic aspects. The second circumstance involves reflection of the existing amount of capital. As we see, this factor of production, in contrast to labor, does not figure in the model in an explicit form.[22] Calculations have shown that, if the amount of capital is taken into account, some of the model's estimated parameters become statistically insignificant. Therefore, it is desirable to limit ourselves to just one basic factor: labor. What is more, even if there is no strictly econometric problem, it is justified to consider only labor as the main factor determining the potential output level. The point is that for the U.S. economy (and not only for it) labor is in shorter supply than capital. According to various calculations, for the United States the so-called natural capital utilization level is approximately 82 percent,[23] while the natural rate of unemployment is less than 6 percent.

We incorporate (17) into (12), so that

$$Y_i(t) = Y_{pot(i)} f(t_i) = Ae^{\lambda i} L_i^{\mu} L_{i-1}^{\eta} Y_{i-1}^{\theta} (-et_i^{\delta} \delta \ln t_i)\,, \tag{18}$$

and by taking the logarithm we transform this expression into a regression equation with the following form:

$$\ln\left(\frac{Y_i(t)}{-e\ln t_i}\right) = \ln(A\delta) + \lambda i + \mu \ln L_i + \eta \ln L_{i-1} + \theta \ln Y_{i-1} + \delta \ln t_i + \ln \varepsilon_i, \tag{19}$$

where ε_i is a random term characterizing the part of the actual output's deviation from the potential level that is determined by so-called nontax circumstances. The results of estimation of this equation are given in Table 2. As we see, all of the model's estimated coefficients and the absolute term are statistically significant. The regular and adjusted coefficients of determination are highly significant. And there is no autocorrelation problem (it is denied by the Durbin *h*-test, at both the 5 percent and 1 percent significance levels). Consequently, the estimated model is suitable for drawing certain conclusions.

Based on the δ given in Table 2, using equations (14), it is easy to establish that for the period being analyzed:

$t^* = 0.3162,\ \ t^{**} = 0.5856.$

This result indicates several interesting things to us.

First, the derived value of the Laffer fiscal point of the first kind, that is, the optimal tax rate t^* is somewhat higher than the actual value of t for each year over the course of the period under consideration.[24] We can judge the results of the tax burden's deviation from the optimal rate in individual years according to the value of the function $f(t)$. As we have indicated above, $f(t^*) = 1$, and $f(t) < 1$ for any t

Table 2

Results of Estimation of Regression Equation (19) (period analyzed, 1970–2008)

Variables	Coefficients	Estimates	Standard errors	Student's *t*-test	Probability
cons	$\ln(A\delta)$	4.1663	1.1740	3.5486	0.0012
i	λ	0.0168	0.0042	4.0091	0.0003
$\ln L_1$	μ	2.2793	0.5945	3.8342	0.0005
$\ln L_{i-1}$	η	–2.1935	0.5703	–3.8465	0.0005
$\ln Y_{i-1}$	θ	0.4334	0.1259	3.4435	0.0016
$\ln t_i$	δ	0.8685	0.0839	10.3453	0.0000

Notes: $R^2 = 0.9985$, adjusted $R^2 = 0.9982$, $F(4.34) = 4{,}390$, $p < 0.0000$; $DW = 1.5770$, $h = 1.65$.

different from t^*. In the latter case, the actual output level lags behind the potential output level, and the reason for this lag may be either an excessive or insufficient tax burden. At the same time, the more the actual tax rate differs from its optimal value, the greater the difference $1 - f(t)$, that is, the percent difference between the potential and actual outputs. A graphic illustration of this is given in Figure 1, which shows the dynamics of percent values of the lost gross domestic product due to nonoptimality of the tax burden. Figure 1 shows that, according to model (12)–(13), if the Laffer theory is correct, there was a certain resource in the U.S. economy for increasing output by optimizing the tax burden, Because of the low tax burden, this resource was greater than 1 percent in some years (1971, 1975, 1983, 1984, and 2003), and less than 0.2 percent in other years. If we calculate the average value during the period, we find that in 1970–2008 the average annual lag of actual output behind the optimal level due to a nonoptimal tax burden was 0.66 percent. This is a considerable reserve, and therefore it may be said that during the period under consideration, on average, the U.S. economy functioned in conditions of a nonoptimal tax burden.

Second, the values of the Laffer fiscal points of the first and second kinds, t^* and t^{**}, determined according to model (12)–(13) are somewhat higher than the average values for the period of Balatsky's fiscal points of the first and second kinds, t^Y and t^T:

$$\bar{t}^Y = \frac{1}{n}\sum_{i=1}^{n} t_i^Y = 0.2839, \quad \bar{t}^T = \frac{1}{n}\sum_{i=1}^{n} t_i^T = 0.2934,$$

which are determined based on model (1)–(4).[25]

This is to be expected because model (12)–(13) examines the role of taxes from a broader perspective than model (1)–(4). The difference is that Balatsky's point of the first kind t^Y shows what the tax rate should be in order to get the maximum

Figure 1. **Dynamics of Lag of Actual Output Level Behind Optimal Output Level Due to Nonoptimality of the Tax Burden in the United States in 1970–2008**

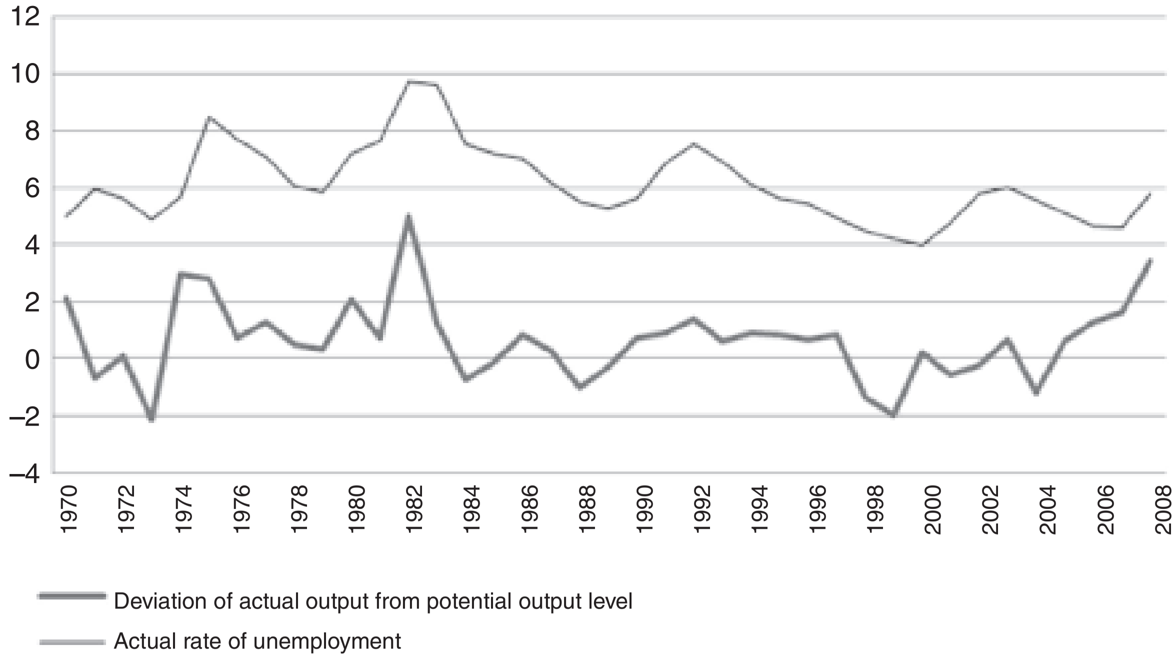
output from the economic resources included in production (those that are actually used), while the Laffer point of the first kind t^* expresses the tax rate at which the maximum output is obtained from existing economic resources (those that can potentially be used). Analogously, Balatsky's point of the second kind t^T is the tax rate corresponding to the maximum budget revenues in conditions of the resources that are already being used, while the Laffer point of the second kind t^{**} is the same thing in conditions of the economic resources that can potentially be used. In other words, the amount of economic resources used for Balatsky's points is given, while the Laffer points should determine this amount themselves.

Third, no less attention should be given to the circumstance that the Laffer fiscal points t^* and t^{**} differ significantly from each other: according to the results obtained, t^{**} is 0.5856, almost twice as great as t^*. At the model level, we can determine what would have happened with the U.S. economy in the period under consideration if the average tax rate had been raised from the actual average value for the period to 0.5856. According to the model, the average tax rate for period $\bar{t} = 0.2772$ corresponds to a lag of approximately 0.7 percentage points behind the potential output. All else being equal, increasing the tax rate to 0.5856 would have increased this lag to 20 percent. This casts doubt on the advisability of an economic policy in which the government's priority is to maximize the budget's tax revenues.

We consider it necessary to make one extremely important clarification. We have in mind that the deviation of the actual output from the potential output may be caused by the effect of a nonoptimal tax burden or by other, nontax circumstances and factors. The function $(1 - f(t))$ reflects only the part of the deviation that is due to the tax burden. The rest of the deviation, which is due to nontax circumstances and factors, is characterized by the random term ε in equation (19). The effect of these circumstances and factors on the amount of output is sometimes more substantial than the effect of the tax burden, and they may work in completely opposite directions. This is confirmed by Figure 2, which shows the dynamics of percent values of the overall deviation of actual output from the potential output estimated according to model (19). As the figure shows, in individual years the deviation from the potential was three or more percentage points, while the maximum deviation because of the nonoptimal tax burden was approximately 1.2 percent. Moreover, in individual years the effect of nontax circumstances was so strong that it exceeded the negative stimuli due to nonoptimality of the tax burden, and the actual output, instead of lagging behind, exceeded the potential output. In Figure 2, such cases correspond to negative values of the deviation.

Along with the curve for the dynamics of deviations of actual output from the potential output, Figure 2 also shows the curve for the dynamics of the actual unemployment rate. As we see, the movements of these two curves are very similar to each other, which indicates that in conditions of high unemployment the lag behind potential output was accordingly high, while in the case of especially low unemployment rate (less than 6 percent), actual output exceeded the potential. This result should be especially emphasized, since in the proposed model neither the

Figure 2. **Dynamics of Deviation of Actual Output from the Potential Output Level and Unemployment in the United States in 1970–2008** (in %)



unemployment rate nor the amount of labor used are entered as exogenous variables. Moreover, model (12)–(13) makes it possible to endogenously estimate the value of the natural rate of unemployment (the level of unemployment existing in the conditions of potential output). To do this we must turn to a version of Okun's equation,[26] which establishes the correspondence between the unemployment rate and the amount of lost gross domestic product:

$$\frac{Y_{pot(i)} - Y_i}{Y_{pot(i)}} = \rho(u_i - u^*), \tag{20}$$

where u is the existing (actual) unemployment rate; u^* is the natural unemployment rate; and ρ is Okun's coefficient. The latter shows the percent change in the lag of actual output behind potential output in the case of a deviation of the actual unemployment rate from its natural rate by one percentage point.[27] It is easy to see that expression (20) describes a static phenomenon (all of the parameters entering into it are for the same period), and therefore we can call ρ a static Okun coefficient.

Introducing the notations $g_{pot} = (Y_{pot} - Y)/Y_{pot}$ and $\rho_0 = -\rho u^*$, we transform (20) into a regression model:

$$g_{pot(i)} = \rho_0 + \rho u_i + \nu_i,\ i = 1, 2, \ldots, n, \tag{21}$$

where ν is a random term. We estimate parameters ρ_0 and ρ so that in (21) the values of the deviation of actual output from potential output determined by equation (19), are considered to be the values of the dependent variable g_{pot}. As a result of these procedures, in our case we get:

$$\underset{}{\hat{g}_{pot}} = \underset{0.9499}{-2.4375} + \underset{(0.1516)}{0.5049}u,\ R^2 = 0.2306;\ F(1.37) = 11.2;\ DW = 1.663,$$

where the standard errors are indicated in parentheses under the coefficients. The estimated equation is statistically significant by all criteria, and therefore, according to the data for the United States in 1970–2008, the Okun coefficient is $\rho = 0.5049$. Consequently, an increase (or decrease) in actual unemployment rate by one percentage point in comparison with its natural rate causes an increase (or decrease) in the amount of lost gross domestic product by an average 0.5 percent. The value of the natural unemployment rate u^* is 4.8274 percent and is derived from the equation $\rho_0 = -\rho u^*$, in which $\rho = 0.5049$, and $\rho_0 = -2.4375$.

The version of Okun's equation (20) differs from the one that is widely used in contemporary macroeconomics textbooks to illustrate the following version of Okun's law:

$$u_i - u_{i-1} = -\beta(g_{yi} - g^*), \tag{22}$$

in which g represents the normal growth rate of GDP (the growth rate corresponding to a constant unemployment rate); g_{yi} is the actual GDP growth rate: $g_{yi} = (Y_i - Y_{i-1})/Y_i$; β is Okun's parameter, which in this case expresses the effect of the amount of deviation of the actual GDP growth rate from the normal rate on the change in the unemployment rate. It is obvious that (22) describes a dynamic phenomenon, since the characteristics entering into it, both of the growth rate and the unemployment rate, express a change in time. Based on this, we can call β a *dynamic Okun coefficient*. The difference in content of the static and dynamic Okun coefficients is obvious; it is also not surprising that there is a quantitative difference between them. We can easily ascertain this if, using the notations $\Delta u_i = u_i - u_{i-1}$ and $\beta_0 = \beta g^*$, we transform (22) into the following regression model:

$$\Delta u_i = \beta_0 - \beta g_{yi} + \nu_i,\ i = 1, 2, \ldots, n,$$

where ν_i is a random term. Estimating the quantities β_0 and β based on data from the same period that was used for model (21), we get:

$$\underset{}{\Delta\hat{u}} = \underset{(0.1401)}{1.2519} - \underset{(0.0392)}{0.4024}g_y,\quad R^2 = 0.7404;\ F(1.37) = 105;\ DW = 1.788.$$

It follows from this that the dynamic Okun coefficient β = 0.4024, and the normal GDP growth $g^* = \beta_0/\beta = 3.111$ percent.[28]

In conclusion, we dwell on one interesting feature of the econometric version of model (12)–(13) that was examined above, and this concerns lag elements. The latter can be used to analyze dynamic processes and determine short-term and long-term characteristics. This is especially true of the mutual influence of labor and total output. For example, the estimated value 2.2793 of parameter μ given in Table 2 expresses the labor elasticity of output for the given period. As we see, labor growth has a positive effect on current output, which is perfectly logical; μ is a short-term elasticity coefficient. It can be shown that, in the conditions of relationship (22), the long-term labor elasticity of output, that is, the equilibrium elasticity coefficient, is determined as $(\eta + \mu)/(1 - \theta)$. Based on the data in Table 2, the numerical value of the latter is 0.1514, and it is less than the short-term elasticity coefficient.

In summary, we note that the model considered here for estimating the effect of the tax burden on the amount of resource use has worked fairly well in regard to data on the U.S. economy. The results obtained are entirely plausible in an economic sense. When different versions of the calculations were carried out, the estimated model as a whole, as well as its parameters, maintained its stability and did not lose its statistical significance in a fairly broad range of changes in the "sample size." It is interesting that, even when the quality of the model deteriorated (the parameters being estimated became statistically insignificant) as a result of excessive reduction of the sample size, the estimates of the fiscal characteristics t^* and t^{**} changed only slightly. Naturally, all of this is not sufficient grounds for drawing final conclusions about the suitability of the suggested model for conducting specific applied calculations. However, we do believe that, after some future improvements and testing of its performance with statistical data from various countries, the suggested approach may prove to be perfectly acceptable for estimating the efficiency of fiscal policy.

Notes

1. E.B. [A.B.] Atkinson and Dzh.E. Stiglits [J.E. Stiglitz], *Lektsii po ekonomicheskoi teorii gosudarstvennogo sektora* [Lectures on Public Economics] (Moscow: Aspekt Press, 1995); Dzh.E. Stiglits, *Ekonomika gosudarstvennogo sektora* [Economics of the Public Sector] (Moscow: MGU INFRA-M, 1997); L.I. Iakobson, *Gosudarstvennyi sector ekonomiki* (Moscow: GU VShE, 2000).

2. E.V. Balatskii [Balatsky], "Analiz vliianiia nalogovoi nagruzki na ekonomicheskii rost s pomoshch′iu proizvodstvenno-institutsional′nykh funktsii," *Problemy prognozirovaniia,* 2003, no. 2, p. 89.

3. Ibid., p. 88.

4. Ibid., p. 89.

5. Several types of equations are used in the practice of mathematical economic modeling, including transformation equations and behavioral equations. A transformation equation describes the correlation between some effect on an object and the result of that effect, in a particular case, the correlation between spending and results. A production function, including (1) is a typical example of such an equation. On the other hand, a behavioral equation characterizes the response of an agent or agents who can make a choice to stimuli or irrational factors (R.L. Raiatskas and M.K. Plakunov, *Kolichestvennyi analiz v ekonomiki* (Moscow: Nauka, 1987), pp. 98–99; L. Iokhansen [Johansen], *Ocherki makroekonomicheskogo plannirovaniia* [Lectures on Macroeconomic Planning], vol. 1 (Moscow: Progress, 1982), pp. 317–30.

6. See, for example, G.B. Kleiner, *Proizvodstvennye Funktsii: Teoriia, metody, primenenie* (Moscow: Finansy i statistika, 1986).

7. In the model analyzed by Balatsky, particular cases of functions (3) and (4) were considered, in which the absolute terms α_0 and β_0 are equal to zero, but both α_2 and β_2 are different from zero.

8. E.V. Balatskii [Balatsky], "Effektivnost′ fiskal′noi politiki gosudarstva," *Problemy prognozirovaniia,* 2000, no. 5.

9. It is easy to see that, in model (1)–(4), in conditions of zero taxation, (i.e., for $t = 0$), the value of output Y is different from zero, and budget revenues T are equal to zero. For the second extreme, when there is 100 percent taxation (i.e., when $t = 1$), both output and budget revenues are different from zero and coincide with each other, while according to the postulates of Laffer theory, the condition $Y(1) = T(1) = Y(0) = T(0) = 0$ should be fulfilled.

10. The intervals of nonnegativity of the coefficients $\alpha(t)$ and $\beta(t)$ for Russia, Sweden, and the United States were calculated according to data given in Balatsky, "Analiz vliianiia nalogovoi nagruzki na ekonomicheskii rost s pomoshch′iu proizvodstvenno-institutsional′nykh funktsii." The data used for Great Britain are from E.V. Balatskii [Balatsky], "Otsenka vliianiia fiskal′nykh instrumentov na ekonomicheskii rost," *Problemy prognozirovaniia,* 2004, no. 4.

11. We believe that, when the condition of nonnegativity of *MPK* and *MPN* is violated in an identified version of a macroeconomic production function of type (1), we are dealing either with incorrect identification of the model or with incorrect specification, and this means that the given mathematical construct is not applicable to modeling of the technology corresponding to the specific data at the researcher's disposal.

12. We should point out that in Balatskii, "Analiz vliianiia nalogovoi nagruzki na ekonomicheskii rost s pomoshch′iu proizvodstvenno-institutsional′nykh funktsii," for the British economy, the set of values of the average tax rate corresponding to nonnegative values of the marginal products has a completely different form: $0.57 \le t \le 0.92$. It is clear that we are dealing with a different extreme: for simultaneous fulfillment of the conditions $MPK(t) \ge 0$ and $MPN(t) \ge 0$ there has to be an unrealistically high tax burden. At the same time, the actual value of t in 1983–99 was much less than 0.57 (the average value of t in this period was 0.363).

13. Balatskii, "Analiz vliianiia nalogovoi nagruzki na ekonomicheskii rost s pomoshch′iu

proizvodstvenno-institutsional′nykh funktsii"; Balatskii, "Otsenka vliianiia fiskal′nykh instrumentov na ekonomicheskii rost."

14. In 1986–2000, the average annual aggregated tax rate in the United States varied in the range of 0.27–0.31.

15. The Cobb–Douglas function is a good tool for theoretical analysis; however, practice shows that, even with constant elasticity coefficients, in the conditions of specific data it is often unsuitable for modeling production technology.

16. Dzh. Saks [J. Sachs] and F.B. Larren [Larrain], *Makroekonomika. Global′nyi podkhod* [Macroeconomics in the Global Economy] (Moscow: Delo, 1996), p. 247.

17. In the general case, these points are different from Balatsky's points of the first and second kinds. This will be discussed below in detail.

18. It should be noted that the entropy function was first used for illustrating the Laffer theory by V. Papava (see Vladimer Papava, "The Georgian Economy: From 'Shock Therapy' to 'Social Promotion,'" *Communist Economies and Economic Transformation,* 1996, vol. 8, no. 2; V.G. Papava, "Lafferov effekt s posledeistviem," *Mirovaia ekonomika i mezhdunarodnoye otnoseniia,* 2001, no. 7). This model was later generalized by G. Loladze (G.O. Loladze, "O nekotorykh aspektakh krivoi Laffera," *Mikro-makro ekonomika*, 2002, no. 9 [in Georgian]).

19. Unfortunately, in some of our articles (Iu. Ananiashvili and V. Papava, "Laffero-keinsianskii sintez i makroekonomicheskoe ravnovesie," *Obshchestvo i ekonomiki,* 2010, no. 9; Iu. Ananiashvili and V. Papava, "Modeli otsenki vliianiia nalogov na rezul′taty ekonomicheskoi deiatel′nosti," *Ekonomika. Nalogi. Pravo,* 2010, no. 2) there is a technical error: in the equation corresponding to the maximum budget revenues, parameter δ is omitted, and instead of the expression $T(t^{**}) = \delta Y_{pot}/(1 + \delta)$, we gave $T(t^{**}) = Y_{pot}/(1 + \delta)$. This error led to inaccuracy in a small part of the text.

20. In practice, several approaches are used for estimating the potential output level. For a general survey of these approaches, see Frederic S. Mishkin, "Estimating Potential Output," Remarks at the Conference on Price Measurement for Monetary Policy, Federal Reserve Bank of Dallas, Dallas, Texas, May 24, 2007, www.federalreserve.gov/newsevents/speech/mishkin20070524a.htm. Some specific methods for estimating the potential output level are suggested in Iu. Ananiashvili, "Ekonometricheskaia otsenka potentsial′nogo valovogo vnutrennego produkta i estestvennogo urovnia bezpabotnitsy ekonomiki Gruzii," *Ekonomika i biznes,* 2010, no. 5 (in Georgian); E.V. Balatskii [Balatsky], "Otsenka ob"ema potentsial′nogo VVP," *Problemy prognozirovaniia,* 2000, no. 1.

21. The data are taken from the official Web site of the U.S. Bureau of Economic Analysis, www.bea.gov.

22. Although it is indirectly manifested in certain ways in the elements A and Y_{i-1}.

23. "Rossiiskaia Federatsiia. Otdel′nye Voprosy," Doklad Mezhdunarodnogo Valiutnogo Fonda po Rossiskoi Federatsii, no. 05/379, October 2005, p. 8, www.imf.org/external/pubs/ft/scr/2005/rus/cr05379r.pdf.

24. For reference, the actual values of t in 1970–2008 satisfied the inequality $0.2605 \leq t \leq 0.3028$, and the average value of $\bar{t}$ for the period was 0.2772.

25. Balatskii, "Analiz vliianiia nalogovoi nagruzki na ekonomicheskii rost s pomoshch′iu proizvodstvenno-institutsional′nykh funktsii."

26. *Makroekonomika*, ed. L.S. Tarasevich (St. Petersburg: GUEF, 1999), p. 197.

27. ρ is not the unemployment elasticity coefficient of lost gross domestic product.

28. These results are completely consistent with the data given, for example, in Olivier Blanchard's well-known macroeconomics textbook (see O. Blanchard, *Macroeconomics,* 5th ed. (Upper Saddle River, NJ: Pearson Prentice Hall, 2009), pp. 184–85). Note that in some economics textbooks, including those on macroeconomics, the results obtained from the dynamic version of Okun's equation (22) are presented without any qualifications as the results of the static version of Okun's equation (20), which is incorrect, in our opinion.